## Quadrilaterals

Geometry 8

- This Slideshow was developed to accompany the textbook
- Larson Geometry
- By Larson, R., Boswell, L, Kanold, T. D., \& Stiff, L.
- 2011 Holt McDougal
, Some examples and diagrams are taken from the textbook.

Slides created by<br>RichardWright, Andrews Academy<br>rwright@andrews.edu

### 8.1 Find Angle Measures in Polygons

, Polygon

- Closed figure made of straight segments
- Diagonal
- Segment that joins nonconsecutive vertices


Notice that the pentagon is made into 3 triangles.

### 8.1 Find Angle Measures in Polygons

- All polygons can be separated into triangles
, The sum of the angles of a triangle is $180^{\circ}$

- For the pentagon, multiply that by 3


## Polygon Interior Angles Theorem

Sum of the measures of the interior angles of a $n$-gon is ( $n-$ 2) $180^{\circ}$

$$
S=(n-2) \cdot 180^{\circ}
$$

Sum of the measures of the interior angles of a quadrilateral is $360^{\circ}$

### 8.1 Find Angle Measures in Polygons

- The coin is a regular II-gon. Find the sum of the measures of the interior angles.
- The sum of the measures of the interior angles of a convex polygon is $1440^{\circ}$. Classify the polygon by the number of sides.

$$
\begin{aligned}
& S=(n-2) 180^{\circ} \\
& S=(11-2) 180^{\circ}=1620 \\
& 1440^{\circ}=(n-2) 180^{\circ} \\
& 8=n-2 \\
& n=10
\end{aligned}
$$

### 8.1 Find Angle Measures in Polygons

- Find $m \angle T$


$$
\begin{aligned}
& S=(n-2) 180^{\circ} \\
& S=(5-2) 180^{\circ}=540^{\circ} \\
& 93^{\circ}+156^{\circ}+85^{\circ}+x+x=540^{\circ} \\
& 334+2 x=540 \\
& 2 x=206 \\
& x=103
\end{aligned}
$$

### 8.1 Find Angle Measures in Polygons

## Polygon Exterior Angles Theorem

Sum of the measures of the exterior angles of a convex polygon $360^{\circ}$

- What is the measure of an exterior angle of a regular pentagon?

- What is the measure of an interior angle of a regular pentagón?
- 510 \#2-34 even, 40-46 even $=21$

$$
\begin{aligned}
\frac{360}{5} & =72^{\circ} \\
180 & =x+72 \\
x & =108
\end{aligned}
$$

Or

$$
\begin{gathered}
S=(5-2) \cdot 180 \\
S=540 \\
\text { int angle }=\frac{540}{5}=108
\end{gathered}
$$

## Answers and Quiz

- 8.I Answers
- 8.I Homework Quiz


### 8.2 Use Properties of Parallelograms

- On scrap paper draw two sets of parallel lines that intersect each other.
> Measure opposite sides. How are opposite sides related?
- Measure opposite angles. How are opposite angles related?


### 8.2 Use Properties of Parallelograms

- Definition of parallelogram
- Quadrilateral with opposite sides parallel


Opposite sides of parallelogram are congruent
Opposite angles of a parallelogram are congruent

Theorems were demonstrated in the focus

### 8.2 Use Properties of Parallelograms

Consecutive angles in a parallelogram are supplementary

- Remember from parallel lines (chapter 3) that consecutive interior angles are supplementary
Diagonals of a parallelogram bisect each other
- Draw diagonals on your parallelogram
- Measure each part of the diagonals to see if they bisect each other.


### 8.2 Use Propertie\$ Of Parallelograms

, Example:

- Find $x, y$, and $z$ if the figure is a parallelogram.

$$
\begin{aligned}
>x & =70 \\
>y & =42 \\
>z & =20
\end{aligned}
$$

### 8.2 Use Properties of Parallelograms

- Find NM
- Find $\mathrm{m} \angle \mathrm{JML}$

- Find $\mathrm{m} \angle \mathrm{KML}$
, 518 \#4-28 even, $32,36,43,44,46-56$ even $=23$
- Extra Credit 52I \#2, 4 = +2


## Answers and Quiz

- 8.2 Answers
- 8.2 Homework Quiz


### 8.3 Show that a Quadrilateral is a Parallelogram

Review
, What are the properties of parallelograms?

- Opposite sides parallel
- Opposite sides are congruent
- Opposite angles are congruent
- Diagonals bisect each other


### 8.3 Show that a Quadrilateral is a Parallelogram

- If we can show any of these things in a quadrilateral, then it is a parallelogram.
- If both pairs of opposite sides of a quad are parallel, then it is a parallelogram (definition of parallelogram)
- If both pairs of opposite sides of a quad are congruent, then it is a parallelogram.
- If both pairs of opposite angles of a quad are congruent, then it is a parallelogram.
- If the diagonals of a quad bisect each other, then it is a parallelogram.
- If one pair of opposite sides of a quad is both parallel and congruent, then it is a parallelogram.


### 8.3 Show that a Quadrilateral is a Parallelogram

, Examples: Is it a parallelogram?


First: yes
Second: No, congruent is not same as parallel

### 8.3 Show that a Quadrilateral is a Parallelogram

- In quadrilateralWXYZ, $\mathrm{m} \angle \mathrm{W}=42^{\circ}, \mathrm{m} \angle \mathrm{X}=138^{\circ}, \mathrm{m} \angle \mathrm{Y}=42^{\circ}$. Find $m \angle Z$. Is $W X Y Z$ a parallelogram?
- Find $x$ so that MNPQ is a parallelogram.


Sum of angles $=360$

$$
\begin{gathered}
42^{\circ}+138^{\circ}+42^{\circ}+m \angle Z=360^{\circ} \\
m \angle Z=138^{\circ}
\end{gathered}
$$

Diagonals bisect each other

$$
\begin{gathered}
2 x=10-3 x \\
5 x=10 \\
x=2
\end{gathered}
$$

## CONCEPT SUMMARY

## Ways to Prove a Quadrilateral is a Parallelogram

1. Show both pairs of opposite sides are parallel. (Definition)

2. Show both pairs of opposite sides are congruent. (THEOREM 8.7)

3. Show both pairs of opposite angles are congruent. (THEOREM 8.8)

4. Show one pair of opposite sides are congruent and parallel. (Theorem 8.9)

5. Show the diagonals bisect each other. (Theorem 8.10)

v 526 \#4-30 even, 34, 36, 39, 43-47 all = 22

## Answers and Quiz

8.3 Answers

- 8.3 Homework Quiz


### 8.4 Properties of Rhombuses, Rectangles, and Squares

- All of these are parallelograms
- Rhombus
- Four $\tilde{=}$ sides
> Rectangle
- Four right $\angle s$
- Square
- Rhombus and Rectangle

- Four $\tilde{=}$ sides
- Four right $\angle \mathrm{s}$



### 8.4 Properties of Rhombuses, Rectangles, and Squares

## Parallelograms

(opposite sides are parallel)
( $4 \cong$ sides)


Squares
$\square$


## Rhombuses

>

### 8.4 Properties of Rhombuses, Rectangles, and Squares

- For any rectangle EFGH, is it always or sometimes true that $\overline{F G} \cong \overline{G H}$ ?
- A quadrilateral has four congruent sides and angles. Classify the figure.

Sometimes, $\overline{F G}$ and $\overline{G H}$ are consecutive sides, not opposite

Square; each angle is $360 / 4=90$ and all sides $\cong$

### 8.4 Properties of Rhombuses, Rectangles, and Squares

- Diagonals

Rhombus: diagonals are perpendicular


Rhombus: diagonals bisect opposite angles


Rectangle: diagonals are congruent


### 8.4 Properties of Rhombuses, Rectangles, and Squares

- ABCD is a rhombus
v Find $m \angle A E D$
, Find DB

- Find AC

Diagonals are perpendicular: $\mathrm{m} \angle \mathrm{AED}=90^{\circ}$

Diagonals bisect each other: $\mathrm{DE}=\mathrm{EB}=8$
$D E+E B=D B=16$

Right triangle $A E B$ is formed with $E B=8$

$$
\begin{aligned}
& \text { Use } \tan 53^{\circ}=\frac{8}{A E} \\
& A E \tan 53^{\circ}=8 \\
& A E=\frac{8}{\tan 53^{\circ}}=6.0
\end{aligned}
$$

### 8.4 Properties of Rhombuses, Rectangles, and Squares

- QRST is a rectangle with $\mathrm{QS}=10$
- Find $m \angle Q P R$
- Find RP

- Find RS
, 537 \#2-52 even, 60-70 even $=32$
- Extra Credit 540 \#2, $5=+2$

$$
\begin{gathered}
m \angle Q P R=m \angle S P T \\
m \angle P T S=m \angle T S P=34^{\circ} \\
m \angle Q P R=m \angle S P T=180^{\circ}-34^{\circ}-34^{\circ}=112^{\circ}
\end{gathered}
$$

Diangonals are congruent and bisect each other

$$
R P=\frac{1}{2} R T=\frac{1}{2} Q S=\frac{1}{2}(10)=5
$$

RT = 10
$m \angle R T S=34^{\circ}$

$$
\begin{gathered}
\sin 34^{\circ}=\frac{R S}{10} \\
R S=10 \cdot \sin 34^{\circ}=5.6
\end{gathered}
$$

## Answers and Quiz

- 8.4 Answers
- 8.4 Homework Quiz


### 8.5 Use Properties of Trapezoids and Kites

- Trapezoid
- Quadrilateral with exactly one pair of parallel sides

- If the legs are $\tilde{=}$, then the trap is isosceles


### 8.5 Use Properties of Trapezoids and Kites

If isosceles trapezoid, then each pair of base angles is $\cong$.


If isosceles trapezoid, then diagonals are $\cong$.
, The converses are also true


### 8.5 Use Properties of Trapezoids and Kites

, Midsegment of a Trapezoid

- Segment connecting the midpoints of each leg


Midsegment Theorem for Trapezoids
The midsegment of a trapezoid is parallel to the bases and its length is the average of the lengths of the bases.

$$
M N=\frac{1}{2}\left(b_{1}+b_{2}\right)
$$

### 8.5 Use Properties of Trapezoids and Kites

- If $\mathrm{EG}=\mathrm{FH}$, is trapezoid EFGH isosceles?

- If $\mathrm{m} \angle \mathrm{HEF}=70^{\circ}$ and $\mathrm{m} \angle \mathrm{FGH}=110^{\circ}$, is trapezoid EFGH isosceles?

Yes, If the diagonals are $\cong$, then the trapezoid is isosceles

$$
\begin{gathered}
m \angle H E F+m \angle E H G=180^{\circ} \\
70^{\circ}+m \angle E H G=180^{\circ} \\
m \angle E H G=110^{\circ}=m \angle F G H
\end{gathered}
$$

Since the base angles are $\cong$, the trapezoid is isosceles

### 8.5 Use Properties of Trapezoids and Kites

- In trapezoid JKLM, $\angle \mathrm{J}$ and $\angle \mathrm{M}$ are right angles, and $\mathrm{JK}=9 \mathrm{~cm}$. The length of the midsegment $\overline{N P}$ of trapezoid JKLM is 12 cm . Find ML.

$$
\begin{gathered}
\text { midsegment }=\frac{1}{2}\left(b_{1}+b_{2}\right) \\
12=\frac{1}{2}(M L+9) \\
24=M L+9 \\
M L=15
\end{gathered}
$$

8.5 Use Properties of Trapezoids and Kites
> Kites


- Quadrilateral with 2 pairs of consecutive congruent sides

If kite, then the diagonals are perpendicular.


If kite, then exactly one pair of opposite angles are congruent.

### 8.5 Use Properties of Trapezoids and Kites

- In a kite, the measures of the angles are $3 \mathrm{x}^{\circ}, 75^{\circ}, 90^{\circ}$, and $120^{\circ}$. Find the value of $x$.
> 546 \#4-32 even, 38, 44-48 all = 21

$$
\begin{gathered}
3 x^{\circ}+75^{\circ}+90^{\circ}+120^{\circ}=360^{\circ} \\
3 x^{\circ}+285^{\circ}=360^{\circ} \\
3 x^{\circ}=75^{\circ} \\
x=25
\end{gathered}
$$

## Answers and Quiz

- 8.5 Answers
- 8.5 Homework Quiz


### 8.6 Identify Special Quadrilaterals

## Quadrilateral



Rectangle


Rhombus


### 8.6 Identify Special Quadrilaterals

- Quadrilateral DEFG has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?
- Give the most specific name for the quadrilateral.


Parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid

Trapezoid (exactly one pair of parallel sides, diagonals not $\cong$ )

### 8.6 Identify Special Quadrilaterals

- Give the most specific name for the quadrilateral.

- A student knows the following information about quadrilateral MNPQ: $\overline{M N} \| \overline{P Q}, \overline{M P} \cong \overline{N Q}$, and $\angle P \cong \angle Q$. The student concludes that MNPQ is an isosceles trapezoid. Why is this wrong?
- 554 \#3-I2 all, I4-30 even, 38, 40, 44-50 even $=25$
$\rightarrow-$ Extra Gredit-557 \#2, $4=+2$

Quadrilateral (not enough information to be more specific)

Could be a square or a rectangle since you don't know the relationship between $\overline{M Q}$ and $\overline{N P}$.

## Answers and Quiz

- 8.6 Answers
- 8.6 Homework Quiz


## 8.Review

- 564 \#|- $/ 8$ all $=18$


## CIAPIERTEST


4. In $\square E F G H, m \angle F$ is $40^{\circ}$ greater than $m \angle G$ Shetch $\square E F G H$ and label each angle with its correct angle measure. Explain your reasoning.
Are you given enough information to determine whether the quadrilateral


In Exercises 8-11, list each type of quadrilateral-parallelogram,
rectangle, rhombus, and square-for which the statement is always true.
a. It is equilateral
5. Its interior angles are all right angles
10. The diagonals are congruent. 11. Opposite sides are parallel
12. The vertices of quadrilateral $P Q R S$ are $P(-2,0), Q(0,3), R(6,-1)$, and $S(1,-2)$. Draw $P Q R S$ in a coordinate plane. Show that it is a traperoid.
13. One side of a quadrilateral /KIM is longer than another side
a. Suppose /KLM is an isosceles trapezold. In a coordinate plane, find posifle coordinates for the vertices of /KCM. Aurtify your answer
b. Suppose /KZM is a kite. In a coordinate plane, find possible
coordinates for the vertices of /KZIM. /writify your answer.
c. Name other special quadrilaterals that /KZ.M could be.

Give the most specific name for the quadrilateral. Eiplain your reasoning

17. In traperoid $w X Y Z \bar{W} \mid \overline{Y Z}$ and $Y Z=4.25$ centimeters. The In traperoid $w X$ midsegment of trapezoid $W X Y Z$ is 2.75 centimeters long. Find $w X$
18. In GRSTU, $\overline{R S}$ is 3 centimeters shorter than $\overline{57}$. The perimeter of ORSTU is 42 centimeters. Find $R E$ and 5 \%

